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Modeling of splat particle splashing data during thermal spraying with the Burr distribution

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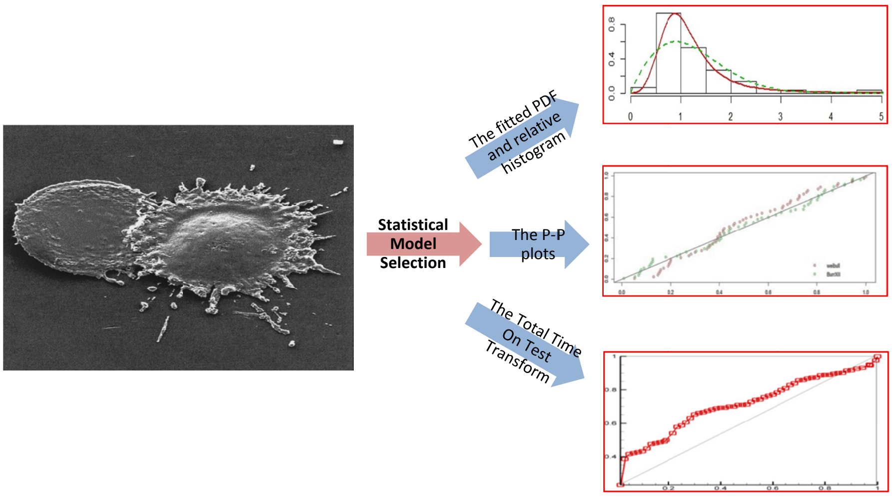
HIGHLIGHTS

- Inference about the splat particle splashing data which sprayed with a normal angle.
- Perform the number of model selection tests to determine the appropriate probability model under complete and progressive censored sample.
- Study of different methods for predicting the missing splat particle splashing data.

GRAPHICAL ABSTRACT



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ABSTRACT

Splashing of splat particles is one of the most important phenomena in industrial processes such as thermal spray coating. The data relative to the degree of splashing of splats sprayed with a normal angle are commonly characterized by the Weibull distribution function. In this present study, an effort has been made to show that the Burr distribution is better than the Weibull distribution for presenting the distribution of the degree of splashing. For this purpose, the Burr Type XII distribution and Weibull distribution are compared using different criteria. Furthermore, because of the great importance of statistical prediction of censored data in reducing costs and improving quality of the coating process, we consider different predictors of this data based on a progressively censored sample. For computing the prediction values we obtain the maximum likelihood estimates using the Expectation-Maximization (EM) algorithm. An important implication of the present study is that the Burr Type XII distribution more appropriately described the degree of splashing data. Therefore, the Burr Type XII can be used as an alternative distribution that adequately describes the splashing data and thereby predicts the censored data.

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1. Introduction

Thermal sprayed coatings are widely used to protect components exposed to corrosion, wear or heat. The mechanical properties of coatings are known to depend strongly on the shape of splat particles formed by individual particles as they impact and freeze. As a good coated surface is extremely important in industry, one of the most important phenomena, due to its impact on the deterioration of the coated surface, is particle splashing. Splashing occurs when a single particle breaks up on impact producing secondary, or satellite, particles. Figure 1 illustrates splashing via a sequence of photographs of the impact of molten tin particles onto a hot surface [1]. Splashing degrade coating quality since they leave voids in the deposit, increasing its porosity and reducing its strength. The physical mechanisms of splashing are still not completely understood and splash study is an extremely interesting and attractive phenomenon. Moreover, prediction of particle splashing can potentially reduce the cost of the development of new coating considerably.

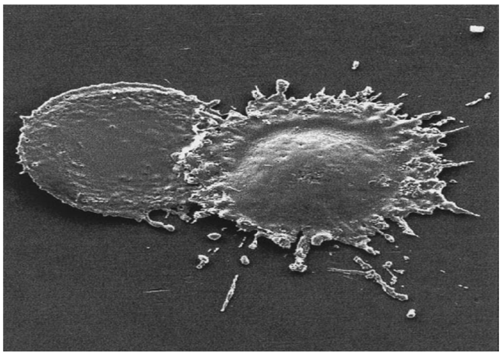

Fig. 1. Splashing of molten tin droplet on a stainless steel surface [1].

The first study of particle fingering and splashing is the remarkable work of Worthington [2,3] published more than a century ago. He drew interesting pictures from direct visual observations of the impact of a mercury drop on a glass plate. More than a half century after Worthington, Engel [4] studied the impact of water droplets onto various surfaces, with application to the erosion of aircraft components due to rain drop impact. A small solid surface roughness has been found to have an important influence on the limiting conditions of the onset of splashing [5]. Montavon et al. [6] studied the effects of spray angle on the morphology of thermally sprayed particles impinging on polished substrates. They evaluated the degree of splashing of splats as a function of their equivalent diameter for 90^{o} and 30^{o} spray angles. Thoroddsen and Sakakibara [7] considered the evolution of the fingering pattern at the edge of drop during the impact of a water drop on a glass plate. They observed that systematic changes in frontal shapes take place during the expansion. Hardalupas et al. [8] examined the impact of a stream of particles onto stainless steel, to examine the influence of surface curvature. Aziz and Chandra [1] studied the impact and solidification of molten tin particles on a stainless steel surface. They photographed particle impact and measured splat diameter and liquid-solid contact angle from these photographs and used a simple energy conservation model to predict the maximum spread of particles during impact.

Asadi et al. [9] extended the numerical and analytical model of the inclined impact of a plasma particle on a solid surface in a thermal spray coating process. The effects of particle velocity, impact angle, and ambient gas pressure (or density) on the threshold of splashing and the motion of the ambient gas surrounding the particle were examined by Liu et al. [10]. Asadi [11] applied a modified computational fluid dynamics and molecular kinetic theory (CFD-MK) method to model the spread and splash of nanoparticle impact on a flat surface. Li et al. [12] estimated impact energy stored in the splash structures via a theoretical model and several morphological parameters. They found that the particle size and the impact velocity displayed similar proportional trends with respect to the splashing height, but did not accompany the secondary particle separation; also the increase of pool temperature dramatically intensified the splashing effect, with the fusiform secondary particle detached from a central jet. Liang et al. [13] examined spreading and splashing processes during a single liquid particle impact on an inclined wetted surface by using a high-speed digital camera. They observed that both surface tension and viscosity can greatly affect the spreading and splashing behaviors. Liang et al. [14] studied rebound and spreading behaviors during a single particle impact on wetted cylindrical surfaces and discussed deformation factor with the critical Weber numbers. While much research has been done on the study of particles splashing, less attention has been paid to the distribution modeling and statistical prediction of this phenomenon.

In the present work, we consider the model selection and prediction of splat particle splashing data obtained by Montavon et al. [6] . We observed that the Weibull distribution has been used as the statistical distribution for modeling the engineering data. We want to answer this question, "Is there a more appropriate statistical distribution?". Thus, we use different methods, such as Kolmogorov-Smirnov (K-S) distance, Akaike information criterion [15], Baysian information criterion [16], and the total time on test (TTT) transform and maximum likelihood criterion, to show the appropriateness of the Burr distribution in the particle splashing data. Since the experimenter may not always obtain complete information on the data in many experimental studies, data obtained from such experiments are called censored data. Type I and Type II censoring schemes are the two most common and popular censoring schemes, but these censoring schemes do not have the flexibility of allowing the removal of units at points other than the terminal point of the experiment. For this reason, in the last few years the progressive censoring scheme has received considerable attention in applied science. This scheme allows one to remove experimental units at points other than the terminal point of the experiment. Several authors have considered different aspects of this censoring scheme; see for example [17-20]. Prediction of censored observation based on the current available data is one of the important problems in engineering experiments. We know that in experiments some of the splashing data are missing. Thus, the second purpose of this paper is to predict future splashing data under the progressive censoring scheme. We obtain the conditional median predictor and prediction interval based on the pivotal method. For obtaining the prediction method, we substitute the unknown parameters with their maximum likelihood estimates under the progressive censoring scheme. It is observed the maximum likelihood estimators (MLE's) cannot be obtained in closed form. So, we propose to use the EM algorithm to compute the maximum likelihood estimators. The EM algorithm is a very powerful and useful tool for analyzing the censored data.

2. Two rival models

In this section we briefly describe Burr Type XII and Weibull distributions as the rival models.

2.1. Burr Type XII distribution

Burr [21] introduced twelve cumulative distribution functions with the primary purpose of fitting distributions to real data. One of the most important of them is the Burr Type XII distribution. The cumulative distribution function and probability density function of the Burr Type XII are given by, respectively

$$F^{(\alpha,\beta)}(x)=\begin{cases}1-(1+x^{\beta})^{-\alpha}; & x\geq 0,\alpha>0,\beta>0\\ 0 & \text{otherwise}\end{cases} \tag{1}$$

$$f^{(\alpha,\beta)}(x)=\begin{cases}\alpha\beta x^{\beta-1}(1+x^{\beta})^{-(\alpha+1)}; & x\geq 0,\alpha>0,\beta>0\\ 0 & \text{otherwise}\end{cases}$$

Here α and β are the two shape parameters. The shape of the hazard rate function of the Burr Type XII distribution depends only on parameter β. For β>0, the hazard rate is eventually decreasing. For β>0, the hazard rate is a unimodal function whereas for β≤0, it is decreasing. Thus the shape β parameter plays an important role in the distribution. Its capacity to assume various shapes often permits a good fit when used to describe biological, clinical, engineering or other experimental data. It also approximates the distributional form of normal, lognormal, gamma, logistic, and several Pearson-Type distributions. For instance, the normal density function may be approximated as a Burr Type XII distribution with β=4.8544 and α=6.2266 and the gamma distribution with shape parameter 16 can be approximated as a Burr Type XII distribution with β=3 and α=6, and the log-logistic distribution is a special case of the Burr Type XII distribution. Extensive work has been done on the Burr Type XII distribution, see for example [22-25].

2.2. Weibull distribution

The Weibull distribution is one of the most popular distributions in analyzing lifetime data. The two parameter Weibull distribution (W) with the shape parameter α>0 and scale parameter β>0 has the probability density function as;

$$f^{(\alpha,\beta)}(x)=\begin{cases}\alpha\beta x^{\beta-1}\exp(-\beta x^{\alpha}); & x\geq 0,\alpha>0,\beta>0\\ 0 & \text{otherwise.}\end{cases} \tag{2}$$

where, α and β are the shape and scale parameters, respectively. If *x~Weibull*(α,β), then the cumulative distribution function, reliability function and hazard function are

$$F^{(\alpha,\beta)}(x)=\begin{cases}1-\exp(-\beta x^{\alpha}); & x\geq 0,\alpha>0,\beta>0\\ 0 & \text{otherwise.}\end{cases}$$

$$R(x)=\exp(-\beta x^{\alpha}),\ \ x\geq 0,\alpha>0,\beta>0$$

and

$$h(x) = \alpha\beta x^{\beta-1},\ \ x \ge 0, \alpha > 0, \beta > 0$$

respectively. A detailed discussion of the W distribution has been provided by [26].

3. Different Criteria for model selection

In this section we describe different available criteria for choosing the best fitted model to a given dataset. Suppose there are two families, say, $F = \left\{ f^{\theta}(.), \theta \in R^{p} \right\} = (f)$ and $G = \left\{ g^{\theta'}(.), \theta' \in R^{q} \right\} = (g)$, the problem is to choose the correct family for a given dataset $\{x_1, ..., x_n\}$. The following criteria can be used for model selection.

3.1. Kolmogorov-Smirnov (K-S) distance

The K-S distance is one of the important distances between two distribution functions, say F and G, and it can be described as follows;

$$D(F,G) = \sup_{-\infty < x < \infty} \left| F(x) - G(x) \right| \tag{3}$$

To implement this procedure, a candidate from each parametric family that has the smallest K-S distance should be found and then the different best fitted distributions should be compared.

3.2. Akaike's information criterion

Consider a sample of independently identically distributed (i.i.d.) random variables, $X_1,...,X_n$ having the probability density function $h(.)=h$. The Kullback-Leibler (KL) information in favor of h against f^{θ} is defined as

$$KL(h, f^{\theta}) = E_h\left(\log \frac{h(X)}{f^{\theta}(X)} \right) = \int_{-\infty}^{\infty} h(x) \log \frac{h(x)}{f^{\theta}(x)}\, dx$$

We have $KL(h,f^{\theta}) \ge 0$ and $KL(h,f^{\theta})=0$, which implies that $h=f^{\theta}$. The KL divergence is often intuitively interpreted as a distance between the two probability measures, but this is not mathematically a distance; in particular, the KL divergence is not symmetric. Akaike [15] introduced the Akaike information criterion (AIC) to select the best model under parsimony. The goal of AIC is to minimize the KL divergence of the selected model from the true model. Notice that the relevant part of the KL divergence is $E_h(\log f^{\theta}(X))$ which has an estimator as

$$\frac{1}{n} \sum_{i=1}^{n} \log f^{\hat{\theta}_n}(x_i) \tag{4}$$

where, $\hat{\theta}_n = (\hat{\alpha}_n, \hat{\beta}_n)$ is the maximum likelihood estimator (MLE) of $\theta = (\alpha,\beta)$. It can be considered as an estimator of the divergence between the true density and the model. Akaike introduced his criterion to model selection as

$$AIC^{f}(\hat{\theta}_n) = -2 \sum_{i=1}^{n} \log f^{\hat{\theta}_n}(x_i) + 2p \tag{5}$$

where, p is the number of parameters in the model. Now choose the family F if $AIC^{f} < AIC^{g}$ otherwise choose family G. For computing the maximum likelihood estimators of unknown parameters of the mentioned distributions (Burr Distribution and Weibull distribution), one can use the inbuilt packages like *nlm*() and *optim*() of the R-software [27].

3.3. Bayesian information criterion

The Bayesian information criterion (BIC) is one of the important criteria for determining the best model for a given data. One major difference of this criterion is the different penalty term that it uses. Thus BIC [16] is defined as

$$BIC^{f}(\hat{\theta}_n) = -2 \sum_{i=1}^{n} \log f^{\hat{\theta}_n}(x_i) + p \log n \tag{6}$$

where, p and n are the number of parameters and sample size, respectively. The BIC is based on Bayesian probability and can be applied to models estimated by the maximum likelihood method. We choose family F if $BIC^{f} < BIC^{g}$; otherwise we choose family G.

3.4. The Total Time On Test (TTT) transform

The total time on test (TTT) transform is a convenient tool for examining the nature of the hazard rate and accordingly checking for the adequacy of a model to represent the failure behavior of the data. The TTT transform of a probability distribution with absolutely continuous distribution function $F(.)$ is given by

$$\varphi_F(x) = H_F^{-1}(x) / H_F^{-1}(1)$$

where, $H_F^{-1}(x) = \int_0^{F^{-1}(x)} \left[1 - F(u) \right] du; \quad 0 \le u \le 1$. The corresponding empirical version of the scaled TTT transform is defined as

$$\varphi(i/n)=\frac{H_n^{-1}(i/n)}{H_n^{-1}(1)}=\frac{\sum_{j=1}^{i}(n-j+1)(x_{j:n}-x_{j-1:n})}{\sum_{j=1}^{n}(n-j+1)(x_{j:n}-x_{j-1:n})};\quad i=1,\ldots,n\ ,\ x_{0:n}=0 \tag{7}$$

It has been shown by Aarset [28] that the TTT transform is convex (concave) if the hazard rate is decreasing (increasing). In addition, for a distribution with a bathtub (unimodal) failure rate the scaled TTT transform is first convex (concave) and then concave (convex). In this example, the scaled TTT transform of the data shows that the empirical hazard function is unimodal.

3.5. Maximum Likelihood Criterion (MLC)

Suppose, $\hat{\theta}_n$ and $\hat{\theta}'_n$ are the MLEs of θ and θ', respectively. The maximum likelihood criterion is defined as

$$T(\hat{\theta}_n,\hat{\theta}'_n)=\sum_{i=1}^{n}\ln\left(\frac{f^{\hat{\theta}_n}(x_i)}{g^{\hat{\theta}'_n}(x_i)}\right) \tag{8}$$

Then we choose (f) or (g) as the preferred model if $T(\hat{\theta}_n,\hat{\theta}'_n)$ is greater than zero or less than zero, respectively.

4. Results and discussion

In this section, we consider the degree of splat particle splashing. A particle splashes when it hits a solid body with adequate rate. Immediately after a molten particle impacts on a surface a skinny liquid film jets out radially from under it. The degree of splat splashing is defined as

$$\text{Degree of splat spalshing}=\frac{S}{12.56637}.R^2 \tag{9}$$

where, S is the area of the selected feature and R is the perimeter to area ratio. The degrees of particle splashing data are reported in different spray angles. We use the data of particle normal impact on a solid surface. The mean, standard deviation and the coefficient of skewness are calculated as 1.2052, 0.7104 and 2.3326, respectively. The measure of skewness indicates that the data are positively skewed. For comparison purposes, we have fitted Burr XII and Weibull distributions to the complete observation. The plot of the empirical and the fitted cumulative distribution functions for these distributions and the fitted probability distribution functions (PDFs) and the relative histogram for the degree of splashing are presented in Figures 2 and 3, respectively. Theses plots indicate that the fitted Burr XII distribution is better than the fitted Weibull distribution. The estimated parameter values, AIC values, BIC values Kolmogorov-Smirnov (K-S) distances and the corresponding p-value are presented in Table 1. From the K-S distances, AIC, BIC values, MLC and p-values of Table 1, it is quite clear that the Burr XII model with estimated parameters provides a much better fit than the Weibull distributions. We also present the percentile-percentile (P-P) plots of the Burr XII and Weibull distributions for the degree of splashing data in Figure 4. This plot shows a strong relationship supporting the appropriateness of the Burr XII distribution. Also, we consider a graphical method based on total time on test (TTT) transform. The plot of the scaled TTT transform of this data set, Figure 5, indicates that the empirical hazard function is unimodal; therefore, it is reasonable to use a BXII distribution to analyze the data.

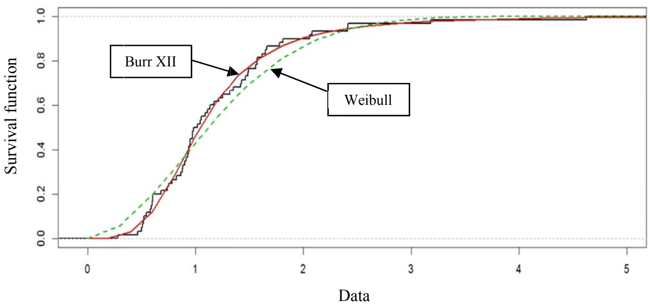

Fig. 2. Empirical function and the fitted functions for degree of splashing.

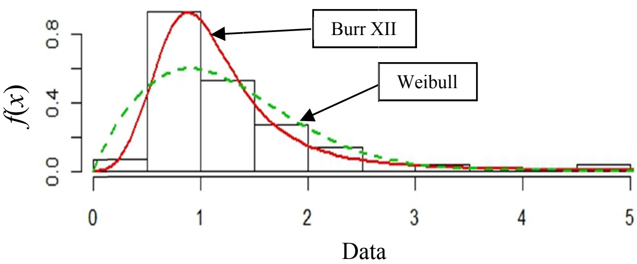

Fig. 3. The fitted PDF and relative histogram for the degree of splashing.

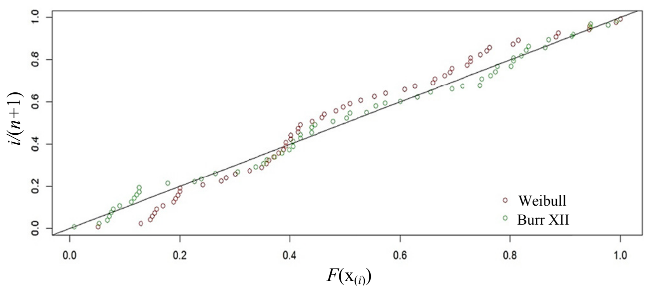

Fig. 4. The P-P plots for degree of splashing.

Table 1. Estimated parameters, K-S distances and AIC values for different distribution functions of the degree of splashing data.

Distribution	Estimated parameters	K-S (p-value)	AIC	BIC	MLC
Weibull	α=0.56168 , β=1.8533	0.1134 (0.4237)	113.6794	117.86810	7.50859
Burr XII	α=0.89478 , β=3.66013	0.0744 (0.8944)	98.4622	102.65090	

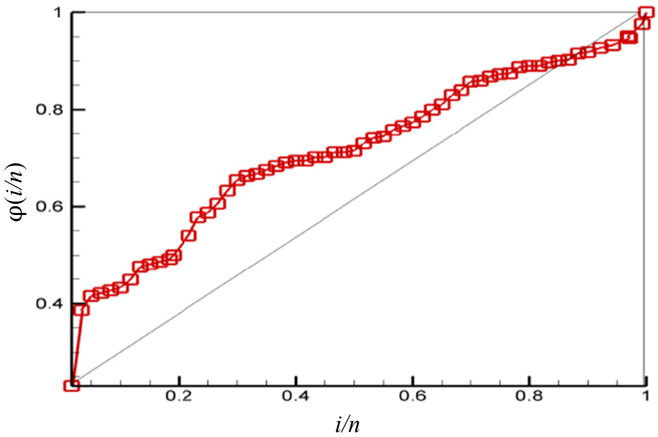

Fig. 5. The scaled TTT transform of degree of splashing.

5. Different Prediction Methods

Often do not have all the data in engineering sciences. In other words, some of the data are missing (censoring). Prediction of censored data based on observed data is a very interesting topic in applied science. We know that the splashing data follows a progressive censoring scheme. This censoring can be described as follows. Consider an experiment in which n units are placed on an experimental test and only $m(<n)$ are completely observed until failure. The censoring occurs progressively in m stages. These m stages offer failure times of the m observed units. At the time of the first failure (the first stage) $X_{1:m:n}$, r_1 of the $n-1$ units are randomly removed (censored) from the experiment. Similarly, at the time of the second failure (the second stage) $X_{2:m:n}$, r_2 of the $n-2-r_1$ units are randomly removed (censored) from the experiment. Finally, at the time of the m^{th} failure (the mth stage) $X_{m:m:n}$, all the remaining $r_m=n-m-(r_1+r_2+\ldots+r_{m-1})$ units are removed from the experiment. We will refer to this as the progressive censoring scheme $(r_1, r_2, \ldots, r_m)$. Now, let $X_{1:m:n}, X_{2:m:n}, \ldots, X_{m:m:n}$ denote a progressively censored sample from the Burr XII model, with $(r_1, r_2, \ldots, r_m)$ being the progressive censoring scheme. For simplicity, we replace $X_{i:m:n}$ by X_i throughout the paper. Our interest is to predict $Z=X_{i,(s)}$; $s=1, 2, \ldots, r_i$; $i=1, 2, \ldots, m$ in all m stages of censoring based on the observed progressively censored sample $x=(x_1, \ldots, x_m)$. For this purpose, first we estimate the unknown parameters of Burr XII distribution using the maximum likelihood method. The likelihood function based on a progressive censored sample from $BXII(\alpha,\beta)$ is given by

$$l(\alpha,\beta\,|\,data) \propto \alpha^m\, \beta^m \prod_{i=1}^{m} x_i^{\beta-1}(1+x_i^{\beta})^{-(\alpha(r_i+1)+1)}$$

and the corresponding log likelihood function is

$$L(\alpha,\beta\,|\,data) = \log l(\alpha,\beta\,|\,data) \propto m\log\alpha + m\log\beta \tag{10}$$

$$+(\beta-1)\sum_{i=1}^{m}\log x_i - \sum_{i=1}^{m}\{\alpha(r_i+1)+1\}\log(1+x_i^{\beta})$$

Taking derivatives with respect to α and β of (10) and putting then equal to zero we obtain

$$\frac{\partial L}{\partial\alpha} = \frac{m}{\alpha} - \sum_{i=1}^{m}(r_i+1)Ln(1+x_i^{\beta}) = 0 \tag{11}$$

$$\frac{\partial L}{\partial\beta} = \frac{m}{\beta} + \sum_{i=1}^{m}\log(x_i) - \sum_{i=1}^{m}\frac{\{\alpha(r_i+1)+1\}x_i^{\beta}\log(x_i)}{1+x_i^{\beta}} = 0 \tag{12}$$

Maximum likelihood estimates of α and β, say $\hat{\alpha}$ and $\hat{\beta}$ respectively, can be obtained by solving these two likelihood equations. But the explicit solutions of (11) and (12) cannot be obtained. We propose to use the EM algorithm to compute the MLEs of the unknown parameters which involves solving two one dimensional optimization problems rather than one two dimensional problem (see Appendix). Now, we want to evaluate the different methods of the prediction of $Z=X_{i,(s)}$; $s=1, 2, \ldots, r_i$; $i=1, 2, \ldots, m$ [29]. We know that the conditional distribution of Z given $\underline{X}$ is just the distribution of Z given $X_i=x_i$ due to the Markovian property of progressively censored order statistics. This indicates that the density of Z given $\underline{X}=\underline{x}$ is the same as the density of the s^{th} order statistic out of r_i units from the left truncated distribution with the density function

$$\hbar^*(z) = \frac{f(z)}{1-F(x_i)}; \qquad z > x_i \tag{13}$$

where

$$f(z) = \alpha\beta z^{\beta-1}(1+z^{\beta})^{-\alpha-1}$$

and

$$F(x_i) = 1-(1+x_i^{\beta})^{-\alpha}$$

Thus, the conditional density of $Z=X_{i,(s)}$ given $X_i=x_i$ for the Burr distribution is given by

$$f(z|data,\alpha,\beta) = \frac{r_i!}{(s-1)!(r_i-s-d)!}\alpha\beta z^{\beta-1}(1+z^{\beta})^{-(\alpha+1)} \tag{14}$$
$$\left[(1+x_i^{\beta})^{-\alpha}-(1+z^{\beta})^{-\alpha}\right]^{s-1}\times(1+z^{\beta})^{-\alpha(r_i-s)}(1+x_i^{\beta})^{\alpha r_i}$$

5.1. Conditional Median Predictor

The median of the distribution of $Z=X_{i,(s)}$ given $X_i=x_i$ whose density is given in (14), is called the conditional median predictor (CMP) [30]. On the other hand, a statistic $\hat{Z}$ is called a conditional median predictor, if

$$P(Z \le \hat{Z}|\underline{X}=\underline{x}) = P(Z \ge \hat{Z}|\underline{X}=\underline{x})$$

So

$$P_{\alpha,\beta}\left(Z \le \hat{Z}|\underline{X}=\underline{x}\right) = P_{\alpha,\beta}\left(1-\left(\frac{1+Z^{\beta}}{1+x_i^{\beta}}\right)^{-\alpha} \ge 1-\left(\frac{1+\hat{Z}^{\beta}}{1+x_i^{\beta}}\right)^{-\alpha}|\underline{X}=\underline{x}\right)$$

It is clear that the distribution of $1-\left(\frac{1+Z^{\beta}}{1+x_i^{\beta}}\right)^{-\alpha}$ given $X_i=x_i$ is a $Beta(s,\ r_i\text{-}s+1)$ distribution with a pdf of

$$f(w) = \frac{w^{d-1}(1-w)^{n-s-d}}{Beta\,(d\,,n-s-d+1)};\quad 0<w<1$$

So, the conditional median predictor can be written as

$$\hat{Z}_{CMP} = \left[\left(\left(1-Med(B)\right)^{-1/\alpha}(1+x_i^{\beta})\right)-1\right]^{\frac{1}{\beta}} \tag{15}$$

Where, B has a Beta distribution with shape parameters s and r_i-s-1, respectively.

5.2. Pivotal Quantity

In this case, our interest is to predict $Z=X_{i,(s)}$ using the pivotal method. So, we choose $W=1-\left(\frac{1+Z^{\beta}}{1+x_i^{\beta}}\right)^{-\alpha}$ as a pivotal quantity for obtaining the prediction interval for Z. Therefore, the $100(1-\wp)\%$ prediction interval for the order statistic Z is given by eq. (16), where, $B_{\wp/2}$ is the percentile of the Beta distribution with parameters s and r_i-s+1, respectively. Prediction interval can be obtained by substituting the unknown parameters with

$$L_{Pivot} = \left[\left(\left(1-B_{\wp/2}\right)^{1/}(1+x_i\ )\right)-1\right]^{-}$$
$$\tag{16}$$
$$U_{Pivot} = \left[\left(\left(1-B_{1-\wp/2}\right)^{1/}(1+x_i\ )\right)-1\right]^{-}$$

their MLEs. Now, we consider the different prediction methods for predicting the censored splashing data. We propose that m=46 and consider the following two censoring schemes:

Censoring Scheme 1: (19*0, 3, 10, 10*0, 1, 14*0). We obtain the following progressively censored sample: 0.2783, 0.4688, 0.5054, 0.5128, 0.5201, 0.5274, 0.5494, 0.5860, 0.5934, 0.6007, 0.6080, 0.6080, 0.6813, 0.7399, 0.7545, 0.7838, 0.8278, 0.8644, 0.8791, 0.8864, 0.9010, C, 0.9304, 0.9377, C ,0.9523, C, 0.9743, 0.9743, 0.9816, 1.0183, 1.0476, 1.0549, 1.0915, 1.1135, 1.1355, 1.1721, 1.2161, 1.2527, C, C, C, 1.4285, C, 1.4871, 1.4945, C, C, C, C, 1.6483, 1.6630, C, 1.8095, 2.0586, C, 2.4102, C, 3.1794, 4.6300.

Censoring Scheme 2: (45*0, 14). We obtain the following progressively censored Sample: 0.2783, 0.4688, 0.5054, 0.5128, 0.5201, 0.5274, 0.5494, 0.5860, 0.5934, 0.6007, 0.6080, 0.6080, 0.6813, 0.7399, 0.7545, 0.7838, 0.8278, 0.8644, 0.8791, 0.8864, 0.9010, 0.9157, 0.9304, 0.9377, 0.9377 ,0.9523, 0.9523, 0.9743, 0.9743, 0.9816, 1.0183, 1.0476, 1.0549, 1.0915, 1.1135, 1.1355, 1.1721, 1.2161, 1.2527, 1.3186, 1.3553, 1.4212, 1.4285, 1.4652, 1.4871, 1.4945, C, C, C, C, C, C, C, C, C, C, C, C, C, C.

Here, C denotes the censored data. In both the schemes we have estimated the unknown parameters using the MLEs. For computing the MLEs we have used the EM algorithm. For schemes 1 and 2, the MLEs of (α,β) for the Burr XII distribution are (1.09106, 3.37267), (1.52112, 4.33760), respectively. The results for different prediction methods and different censoring schemes are presented in Tables 2 and 3. From these tables, it is observed that the prediction methods work well.

6. Conclusion

In this paper, we compare Weibull distribution and Burr XII distribution for the degree of particle splashing

Table 2. Values of interval prediction method for $X_{i,(s)}$; $s=1, \ldots, r_i$, (for scheme 1, $i=20, 21, 32$ and for scheme 2, $i=46$) and their real values.

Scheme 1			Scheme 2		
$X_{i,(s)}$	Real values	PI (Pivot)	$X_{i,(s)}$	Real values	PI (Pivot)
$X_{20,(1)}$	0.9157	(0.8832, 0.9517)	$X_{46,(1)}$	1.5604	(1.5287, 1,5935)
$X_{20,(2)}$	0.9377	(0.9245, 0.9734)	$X_{46,(2)}$	1.5750	(1.5335, 1.5865)
$X_{20,(3)}$	1.3186	(1.2789, 1.3395)	$X_{46,(3)}$	1.5750	(1.5335, 1.5865)
$X_{21,(1)}$	1.3553	(1.3228,1.3497)	$X_{46,(4)}$	1.6190	(1.5758, 1,7169)
$X_{21,(2)}$	0.9523	(0.9410, 0.9844)	$X_{46,(5)}$	1.6483	(1.6355, 1.6697)
$X_{21,(3)}$	1.4212	(1.4052, 1.4637)	$X_{46,(6)}$	1.6630	(1.6067, 0.6954)
$X_{21,(4)}$	1.4652	(1.4180, 1.4909)	$X_{46,(7)}$	1.7802	(1.7525, 1,8240)
$X_{21,(5)}$	1.5604	(1.5356, 1.6011)	$X_{46,(8)}$	1.8095	(1.7843, 1,8278)
$X_{21,(6)}$	1.7802	(1.7525, 1.8240)	$X_{46,(9)}$	2.0586	(2.0168, 2.1137)
$X_{21,(7)}$	1.5750	(1.5199, 1.5747)	$X_{46,(10)}$	2.0805	(1.9956, 2.1754)
$X_{21,(8)}$	2.0805	(1.9956, 2.1754)	$X_{46,(11)}$	2.4102	(2.3768, 2.4096)
$X_{21,(9)}$	1.5750	(1.5199,1.5747)	$X_{46,(12)}$	2.4175	(2.3766, 2.4562)
$X_{21,(10)}$	2.4175	(2.3766, 2.4562)	$X_{46,(13)}$	3.1794	(3.1547, 3.2128)
$X_{32,(1)}$	1.6190	(1.5699, 1.7068)	$X_{46,(14)}$	4.6300	(4.6036, 4.4637)

Table 3. Values of point prediction method for $X_{i,(s)}$; $s=1, \ldots, r_i$, (for scheme 1, $i=20, 21, 32$ and for scheme 2, $i=46$) and their real values.

Scheme 1			Scheme 2		
$X_{i,(s)}$	Real values	PI (CMP)	$X_{i,(s)}$	Real values	PI (CMP)
$X_{20,(1)}$	0.9157	0.9076	$X_{46,(1)}$	1.5604	1.5546
$X_{20,(2)}$	0.9377	0.9265	$X_{46,(2)}$	1.5750	1.5637
$X_{20,(3)}$	1.3186	1.3245	$X_{46,(3)}$	1.5750	1.5637
$X_{21,(1)}$	1.3553	1.3463	$X_{46,(4)}$	1.6190	1.6045
$X_{21,(2)}$	0.9523	0.9511	$X_{46,(5)}$	1.6483	1.6247
$X_{21,(3)}$	1.4212	1.4286	$X_{46,(6)}$	1.6630	1.6739
$X_{21,(4)}$	1.4652	1.4625	$X_{46,(7)}$	1.7802	1.7748
$X_{21,(5)}$	1.5604	1.5587	$X_{46,(8)}$	1.8095	1.8137
$X_{21,(6)}$	1.7802	1.7819	$X_{46,(9)}$	2.0586	2.0455
$X_{21,(7)}$	1.5750	1.5688	$X_{46,(10)}$	2.0805	2.0732
$X_{21,(8)}$	2.0805	2.0654	$X_{46,(11)}$	2.4102	2.4269
$X_{21,(9)}$	1.5750	1.5567	$X_{46,(12)}$	2.4175	2.4020
$X_{21,(10)}$	2.4175	2.4030	$X_{46,(13)}$	3.1794	3.1663
$X_{32,(1)}$	1.6190	1.5941	$X_{46,(14)}$	4.6300	4.6451

in thermal spray. Different plots and statistical criteria were used to identify the best fitted distribution for this data. Using several statistical criteria, like minimum K-S distance, minimum AIC value and minimum BIC value, the Burr XII distribution function appears to be a more appropriate statistical distribution function for this data. One important problem in engineering sciences is the prediction of future observations. So, we reported the different prediction values of future observations and observed that these methods of prediction work well. Finally, we should mention that our results can be extended for the degree of particle splashing observed at other spray angles.

Appendix

The EM algorithm is an efficient iterative procedure to compute the maximum likelihood estimate in the presence of missing data and consists of an expectation step (E-step) and a maximization step (M-step). First, let us denote the observed and censored data by $X = (X_1, ..., X_m)$ and $Z = (Z_1, ..., Z_m)$, respectively, where each Z_j is $1\times r_j$ vector $Z_j = (Z_{j1}, ..., Z_{jr_j})$ for $j=1,..., m$ and they are not observable. The censored data vector Z can be thought of as missing data $W= (X,Z)$ and represents the complete data set. Therefore, the log-likelihood function $L_c= (W; \alpha, \beta)$ of the complete data after ignoring the constants can be written as:

$$L_c\left(W;\alpha,\beta\right) = n\log\alpha + n\log\beta + (\beta-1)\sum_{i=1}^{m}\log x_i - (\alpha+1)\sum_{i=1}^{m}\log(1+x_i^{\beta})$$

$$+(\beta-1)\sum_{i=1}^{m}\sum_{k=1}^{r_i}\log z_{ik} - (\alpha+1)\sum_{i=1}^{m}\sum_{k=1}^{r_i}\log(1+z_{ik}^{\beta}) \quad (17)$$

E-step:

This step involves the computation of the conditional expectation of the log-likelihood with respect to the incomplete data given the observed data. For this purpose, we compute the pseudo log-likelihood function as:

$$L_s\left(\alpha,\beta\right) = E\left(L_c\left(W;\alpha,\beta\right)\middle| X\right) = n\log\alpha + n\log\beta + (\beta-1)\sum_{i=1}^{m}\log x_i$$

$$-(\alpha+1)\sum_{i=1}^{m}\log(1+x_i^{\beta}) + (\beta-1)\sum_{i=1}^{m}\sum_{k=1}^{r_i}\mathrm{E}\left[\log z_{ik}\left|z_{ik} > x_i\right.\right]$$

$$-(\alpha+1)\sum_{i=1}^{m}\sum_{k=1}^{r_i}\mathrm{E}\left[\log(1+z_{ik}^{\beta})\left|z_{ik} > x_i\right.\right] \quad (18)$$

where

$$E\left(\log Z_{ik}\left|Z_{ik} > c\right.\right) = \frac{\alpha\beta}{1-F_X(c;\alpha,\beta)}\int_c^{\infty} x^{\beta-1}\left(1+x^{\beta}\right)^{-(1+\alpha)}(\log x)dx$$

and

$$E\left(\log(1+Z_{ik}^{\beta})\left|Z_{ik} > c\right.\right) = \frac{\alpha\beta}{1-F_X(c;\alpha,\beta)}\int_c^{\infty} x^{\beta-1}\left(1+x^{\beta}\right)^{-(\alpha+1)}$$

$$\log(1+x^{\beta})dx = \alpha^{-1} + \log\left(1+c^{\beta}\right)$$

We denoted $E\left(\log Z_{ik}\left|Z_{ik} > c\right.\right)$ and $E\left(\log(1+Z_{ik}^{\beta})\left|Z_{ik} > c\right.\right)$ by A(c, α, β) and B(c, α, β), respectively.

M-step:

This step includes the maximization of the pseudo log-likelihood function (18). Therefore, if at the k^{th} stage the estimate of (α, β) is $(\alpha^{(k)}, \beta^{(k)})$, then $(\alpha^{(k+1)}, \beta^{(k+1)})$ can be obtained by maximizing

$$L_c^{*}\left(W;\alpha,\beta\right) = n\log\alpha + n\log\beta + (\beta-1)\left[\sum_{i=1}^{m}\log x_i\right] - (\alpha+1)\left[\sum_{i=1}^{m}\log(1+x_i^{\beta})\right]$$

$$+(\beta-1)\sum_{i=1}^{m} r_i A(x_i,\alpha^{(k)},\beta^{(k)}) - (\alpha+1)\sum_{i=1}^{m} r_i B(x_i,\alpha^{(k)},\beta^{(k)}) \quad (19)$$

with respect to α and β. Notice that the maximization of (19) can be obtained by different methods, such as, Kundu and Pradhan [31].

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