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Effects of local vibration on silo discharge and jamming: Employing an experimental approach

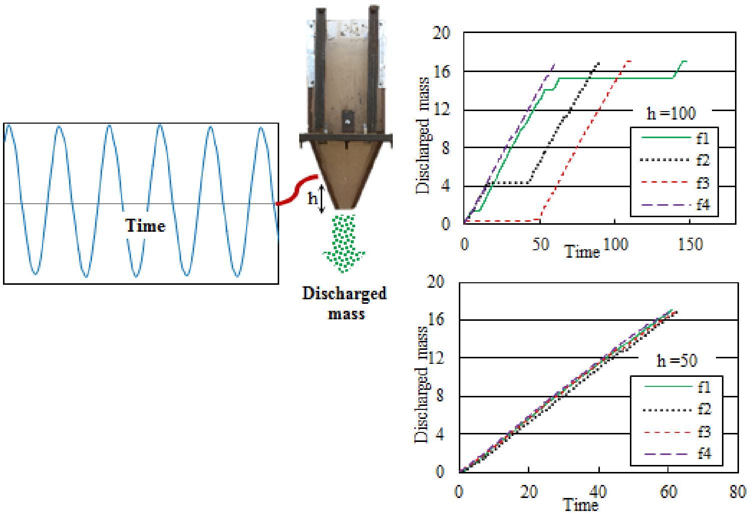
Akbar Jafari*, Ali Abolghandi, Arash Gharibi, Mohammad Vahid Khalili Parizi, Ali Begheri Jamebozorgi

Department of Mechanical Engineering, Sirjan University of Technology, Sirjan, Iran

HIGHLIGHTS

- Local vibration could increase the average discharge rate.
- It was observed that local vibration does not affect instant discharge rate.
- Location of the local vibrator is a key parameter on its anti-jamming efficiency.
- Local vibration does not considerably affect the discharge rate of an unjammed silo.

GRAPHICAL ABSTRACT



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ABSTRACT

Blockage is a common problem in many practical silo applications, and vibration seems to be a practical solution to overcome this problem. An experimental setup was developed to observe the effects of different vibrational parameters on vibrator anti-jamming efficiency. The silo was made of transparent plates to provide the possibility of watching the materials inside it. The outlet mass was recorded on a computer via a weighing load cell. The vibrator was installed at different locations on the silo walls to reveal effects of the vibrator position on its efficiency to prevent jamming. Moreover, relevant tests were conducted to reveal the effects of the vibration frequency. A vibrometer instrument with contacting probe was employed to measure the local vibration characteristics. The measured data was used to identify the vibration dimensionless acceleration. It was concluded that the location of the vibrator significantly affects its anti-jamming ability. Furthermore, it was observed that the vibration frequency and acceleration influence the impact of the vibration to prevent the silo jamming to some extent. It was observed that while the vibration does not influence the instant discharge rate it does considerably affects the average rate.

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* *Corresponding author: Tel.: +9834-42336901 ; Fax: +9834-42336901 ; E-mail address: jafari@sirjantech.ac.ir*

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1. Introduction

A silo is a common device used in several industrial fields such as mineral processing, agriculture, food and pharmacy factories, petro-chemistry and handling of construction materials (e.g. cement and aggregates). Due to their wide range of applications, considerable research has been performed on silos discharge in the literature. It appears that Janssen was among the first scientists who studied the behavior of granules in the silos [1]. Independency of the bottom pressure on the height of the silo filling is one of the most important results of that research. This is known as the Janssen effect, which guarantees the constant flow rate in an hourglass. Such effect is due to stress propagation via the grains, which is different from what happens in fluids. Such behavior is important in designing and construction of silos [2,3]. Walker and later Walters developed approximation equations to evaluate the silo wall stress and pressure [4,5]. Furthermore, the discharging behavior of the silos is another practical topic that is pertinent to both theoretical and practical viewpoints. For example, Fowler and Glastonbury applied an experimental method in order to develop a semi-empirical equation to estimate the flow of granular solids through orifices [6]. According to their results, particles' density, flow area, diameter, shape and size affect the outflow. Afterwards, Beverloo et al. conducted fundamental experiments and semi-analytical studies to predict the discharge rate, and proposed an equation containing the afore-introduced parameters [7]. It is worthwhile to mention that their results consisting of the Beverloo equation are still invoked in this field. Oldal *et al.* employed an experimental method to develop equations able to predict a silos' discharge rate [8]. Essentially, they introduced an extra parameter related to arching of the materials into the Beverloo equation. Uñac *et al.* employed an experimental approach to study the effects of hopper slope, its outlet size and granules diameter on the discharge rate [9]. According to their case studies, the particles' size and hopper angle affect magnitudes of the parameters in the Beverloo equation. Regarding the particle size effect, Hsiau *et al.* employed an experimental approach and showed that the discharge rate does not depend on the particle size monotonically [10,11]. They concluded that when the particles are too fine or too coarse, the discharge rate is smaller than the case of having intermediate particles. Perge *et al.* developed a type of experimental rig to measure the reaction force on the lateral walls of the silo and the force in the outlet region [12]. According to their results, there is no meaningful relationship between the discharge rate and the local stress in the outlet region. Using an experimental approach, Zuriguel *et al.* showed that there is a critical radius (ratio of the outlet size over the particle size) beyond which jamming does not happen [13]. Moreover, they showed that this critical value depends on the particles' shape. In addition to the mentioned experimental studies, theoretical methods such as the discrete element method (DEM) and statistical methods have also been employed to estimate silos discharge rate, pressure distribution and jamming probability [14-17].

As the main target of the current paper is associated with the jamming and vibration effects, a relevant survey is presented here. Studies about the vibration effects can be categorized into three themes. The first is related to the induced vibration and music due to the silo discharge; the second one concerns the characteristics and flowability of the granular materials under vibration. The third one, which is pertinent to industrial scales and is the focus of the present research, is related to the effects of vibration on the silo discharge and jamming. Regarding the first theme, Muite *et al.* investigated the generated sounds and music during the silo discharge [18]. According to that work, pulsating as a result of stick-slip motion of the granular material during the discharge causes an oscillatory motion and generates music in silos. Based on the experimental results reported by Niedostatkiewicz *et al.*, adding an insert inside the silo can eliminate dynamic-acoustic effects during silo discharge [19]. Pertaining to the second theme, Arnold and Kaaden employed a modified form of the well-known Jenike shear test device in order to study the effects of vibration parameters on the friction between particles [20]. According to their results, vibration weakens the friction between walls and particles, and consequently facilitates their motion. Roberts and Scott applied an almostsimilar approach to reveal the effects of vibration on the shear strength of powder materials [21]. They concluded that transverse vibration reduces shear strength; however, preferable frequencies minimize shear strength. Kollmann and Tomas investigated the effects of vibration on the flow properties of two types of particulate solids [22]. They showed that vibration can reduce wall friction and shear strength. Takahashi *et al.* studied the effects of a vibration bed on the motion and velocity of the particles using experimental and theoretical methods [23]. They tried to develop particle motion equations by validating experimental data. Liffman *et al.* studied the effects of horizontal vibration on the convection of the granular materials on a bed and explored size segregation due to the vibration [24]. Concerning the third theme, Suzuki *et al.* developed a semi-empirical formula to estimate the discharge rate of particles from a hopper under vertical vibration [25]. In that study, the whole body of the hopper was shaken using a vibrator. Interestingly, they observed that vibration does not necessarily increase discharge rate. Hunt et al. studied the effects of horizontal vibration on the silos discharge rate [26]. They showed that horizontal vibration of the whole body of the container can increase the discharge rate and decrease the stagnant regions in the hopper. According to their observations, horizontal vibration can influence the first-in last-out problem in case of funnel-flow conditions. Veje and Dimon experimentally studied the effects of vibration on the flow in an hourglass [27]. According to their results, vibration might have a positive or negative impact on the flow rate depending on the flow regime. They concluded that vibration increases the average discharge rate in case of relaxation flow, whereas it decreases the discharge rate in steady flow conditions. Wassgren *et al.* studied the effects of vertical vibration of the whole body of the hopper on its flow rate [28]. They proved that vibration can increase or decrease the discharge rate depending on the vibration characteristics. They attributed the reduction of the discharge rate to the decreasing of the bulk density due to the vertical vibrational acceleration. Using an experimental method, Chen *et al.* proved that the peak vibration velocity is the key factor that affects the flow [29]. According to their results, regardless of the frequency, increasing the peak velocity leads to decreasing the flux which then becomes a constant value asymptotically. It should be noted that the whole body of the container was shaken in their experimental apparatus. Janda *et al.* experimentally studied the effects of vibration on unjamming of hoppers [30]. Based on their results, vibration can break blockage and decrease the critical diameter of hopper outlet. They showed that, although vibration contributes to unjamming, it almost does not affect the discharge rate. Mankoc et al. reported their experimental results about the effects of vertical vibration on the jamming of a flat bottom silo [31]. They stated that the main role of vibration is the breakage of arches after they are formed, but is without considerable effect on the probability of the arch formation. Tomas and Kache employed theoretical (continuum-based) and experimental approaches to study the effects of vibration on the discharge of a hopper containing ultrafine cohesive powder [32]. According to their results, pulsed vibration can prevent jamming. On the other hand, they concluded that continuous operation of a vibrator leads to a decrease in discharge rate. Lozano *et al.* studied the effects of vertical vibration on breakage of the particles arches in a 2-D experimental silo [33]. They put a layer of particles inside the silo to observe the arch creation and breakage. According to their results, vibration acceleration is a key factor that affects the probability of arches breakage. In addition to the experimental method, theoretical methods consisting of continuum mechanics and DEM have also been employed to study the effect of the vibration on silos discharge. For instance, Matchett proposed a theoretical model to study the effects of vibration on stress in the materials inside hoppers [34]. According to this reference, vibration can affect the stress state in such a way that when the hopper wall moves away from the hopper axis, the stress will relax, leading to flow. Fraige et al. employed 2-D DEM to study the effects of vibration parameters on hopper discharge [35]. They found that vibrator location has a significant impact on preventing blockage, concluding that the best position is at a few particle diameters above the outlet. Furthermore, their numerical results showed that increasing the vibration amplitude enhances the discharge rate, whereas the frequency does not have considerable impact. Langston replicated Fraige's study using a 3-D DEM instead and achieved similar results [36].

As can be observed from the above literature review, the effects of vibration on silos discharge and jamming have not been fully understood. To the best of the author's knowledge, the whole body of the container was shaken in the published experimental studies, whereas local vibration is usually used in practical applications. The mechanical energy penetrates the granular materials via the vibration of the silo walls. In reality, we have often encountered a problem with the efficiency of a local vibrator to prevent the silo jamming. As shown in Fig. 1, a vibrator in non-efficient working conditions not only could not prevent jamming but also caused wall

Fig. 1. Unsuccessful anti-gamming and damage of the silo wall due to non-efficient vibration.

damage. Therefore, we implemented an experimental method to reveal the effects of the local vibration on the flow behavior. The obtained results reported in the current study can be valuable in both theoretical and practical viewpoints.

2. Experiments

To perform the experiments, quasi-spherical mineral aggregates, shown in Fig. 2, were used. Such particles are extensively used for making concrete in the construction industry. The bulk density of the particles is approximately with 10% having a diameter of 4-5 mm and the rest having a diameter of 8-10 mm.

The silo was made of PMMA (poly methyl methacrylate) transparent plate with a thickness of 6 mm. Transparency provides the possibility to see the materials behavior inside the silo. As depicted in Fig. 3, the silo with a square cross section was installed on a structure with a gate at the bottom of the silo that could be opened quickly. Several hoppers with different sizes were attached to the container in order to achieve a reasonable size for the silo outlet. Different outlet sizes were used to find the critical size beyond which continuous outflow occurred without jamming. Finally, experimental studies related to the vibration effects were conducted on both jammed and unjammed silos. A vibrator with a rotating unbalance mass of 14.3 g. and a gyration radius 48 mm was used to apply vibration on a wall of the hopper. By changing the vibrator rotation speed, excitation frequency was set at f =15083, f = 25083, f = 2705 and f = 29.16 Hz which are called f_1, f_2, f_3 and f_4, respectively. In addition to studying the

Fig. 2. Quasi-spherical mineral aggregates in scale of around 1/2.

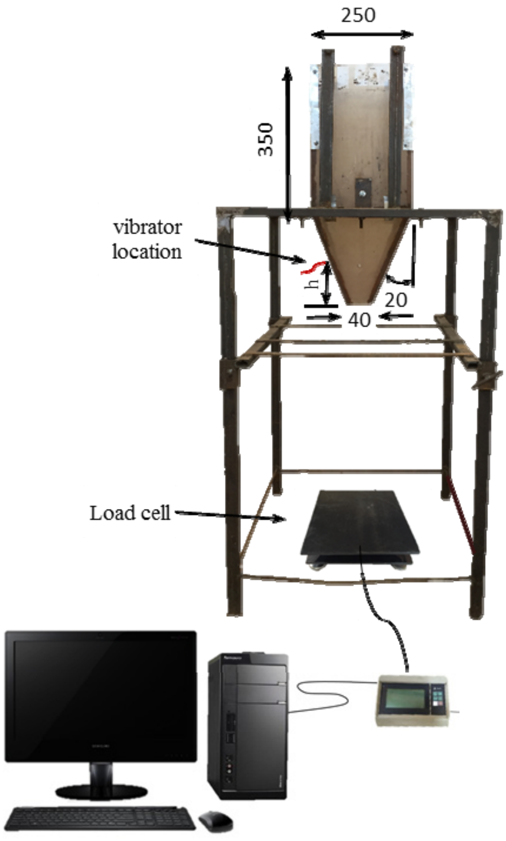

Fig. 3. The silo with transparent walls and the weighing system (numbers are in mm).

frequency effects, the vibrator location was also in scope. Therefore, the vibrator was installed at three different heights, *h*, as indicated in Figures 3 and 4.

To record the mass flow rate, a load cell with the capacity of 100 kg and accuracy of 1 g was used. The load cell was connected to the computer via a RS232 port, and the data versus time were saved every 90 milliseconds.

Vibration force is related to the eccentric mass and rotating frequency. However, vibration amplitude depends on the force magnitude and the vibrator location. Here, a vibrometer instrument (X-viber, Masibus Co.) was used to measure vibration characteristics including the local amplitude. For this purpose, the probe of the vibrometer was fixed on the silo wall, exactly adjacent to the vibrator. The variation of amplitude versus time for a typical experiment condition is shown in Fig. 5. The local vibration amplitude peak at the probe location can be obtained using such data.

3. Results and discussion

Several tests were conducted under various vibration settings when the silo, without vibration, was not jammed. In this situation no considerable deviation was observed between the discharge rates belonging to different vibration conditions. In other words, in the case of having mass flow without obstruction, the vibrator did not considerably influence the silo discharge rate. Therefore, employing local vibration is not suggested for such conditions, because it seems that the silo is working under optimum state. However, more discussions about such a conclusion and comparison with published references will be presented at the end of this section.

Subsequently, several tests under different vibration settings were conducted when the silo was jammed in the absence of the vibrator. In this case, significant deviations were observed between the results related to different vibration conditions. It should be noted that to ensure attaining accurate results, the test relevant to each setting was repeated four times. Variations of the discharged mass versus time for three locations of h =50, h =100, h =150 mm were drawn in Fig. 6. For each vibrator location, the graphs were plotted for four vibration frequencies. The plateau sections in the graphs indicate the intervals of flow interruption due to the jamming problem. Interestingly, it is observed that the graphs that do not have interruption regions are close together, as seen in Fig. 6(a). This means that when the vibrator was installed near the outlet (mm) it worked efficiently to prevent the jamming problem. On the other hand, there are wide deviations between the graphs that have interruption regions (Figures 6(b) and 6(c)) meaning lessening of the vibrator efficiency. To provide a better illustration about the results, more discussions are presented in the coming subsections.

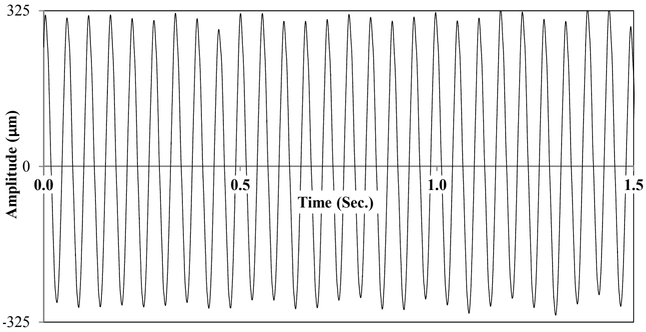

Fig. 5. Recorded local vibration amplitude via vibrometer instrument.

3.1. Anti-jamming and average discharge rate

According to the graphs in Fig. 6, for cases where the vibrator is at the lowest location close to the outlet, there is no blockage and the trend is linear for all frequencies.

$h = h_1 = 50$ mm $h = h_2 = 100$ mm $h = h_3 = 150$ mm

Fig. 4. Vibrator installed at three locations of the hopper wall.

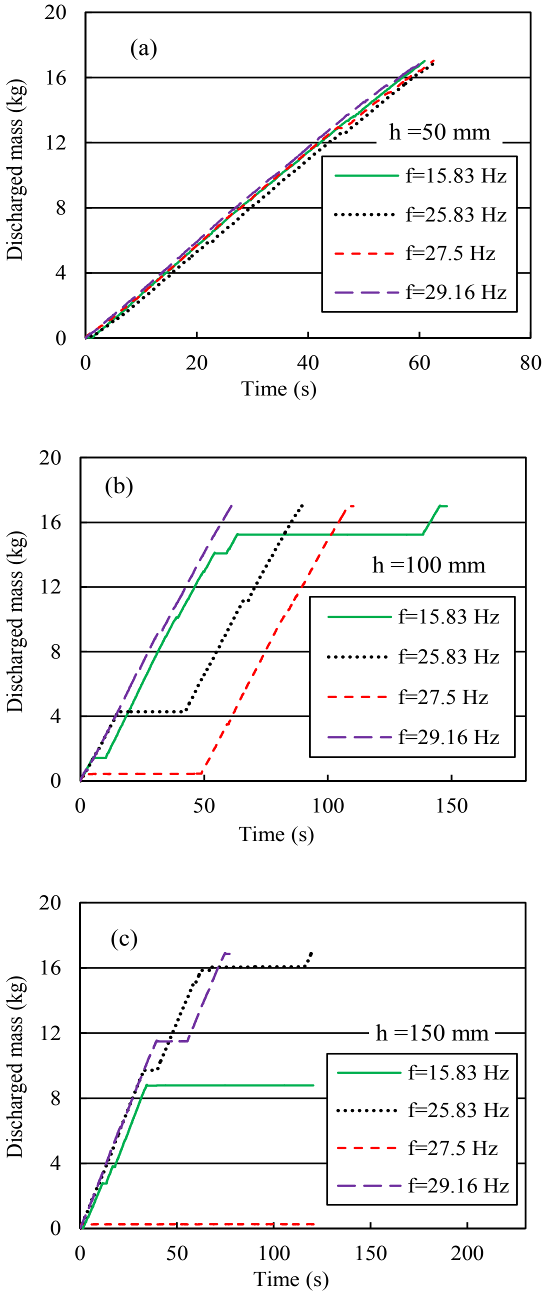

Fig. 6. Discharged mass versus time for different frequencies of the vibrator located at (a) *h* = 50 mm, (b) *h* = 100 mm, and (c) *h* = 150 mm.

On the other hand, it is observed that when the vibrator moves farther from the outlet to higher locations, the total time for the silo to empty is increased. In fact, when the vibrator is moved to higher locations, the number of interruptions in the outflow increased. To observe the impact of the vibrator location more clearly, the total number of interruptions are depicted in Table 1. It is observed that for cases where the vibrator is close to the outlet (h =50 mm), no interruption occurred for h =150 mm all vibration settings. On the other hand, when the vibrator is moved away from the outlet, the number of interruptions increased so that for , the vibrator became inefficient to solve the jamming problem, i.e. the blockage remained. The total discharge times related to different conditions are presented in Fig. 7. It can be observed that when the vibrator was located close to the outlet, the discharge time was approximately 62 seconds regardless of the vibration frequency; whereas, in the cases that the vibrator was moved higher, the discharge time increased to 146 seconds with this time gradually becoming infinite (represented by Inf. in tables and figures), meaning full jamming.

Regarding the frequency effects, it is noted that in cases where the vibrator is close to the outlet, it has the highest efficiency so that no blockage exists, and discharge times pertaining to all frequencies are almost equal. On the other hand, when the vibrator was moved to a higher location its efficiency to prevent blockage is reduced. However, it is observed that increasing the frequency aiding the effect so that the overall discharge time was reduced. When the vibrator was moved much farther from the outlet, the vibrator efficiency reduced even more. In this case, as shown in Table 1 and Figures 6(c) and 7, at the lowest frequency the jamming problem was not solved; however, at the highest frequency the vibrator once again became efficient so that blockage was prevented. In sum, it can be concluded that increasing the frequency enhances a vibrator's ability to solve the jamming problem eve if it is located far from the outlet. Furthermore, the success probability decreases as the frequency decrease until it finally becomes inefficient for the lowest frequency.

3.2. Instant discharge rate

In the previous subsection, the impact of vibrator on the blockage was studied; however, its influence on the instant discharge rate is questionable. In fact, instant discharge rate is calculated as the slope of the mass-time graph, while the average rate is related to the total

Table 1. Number of interruptions for different locations and frequencies of the vibrator.

Vibrator location (mm)	Frequency (Hz)			
	15.83	25.83	27.5	29.16
h =150	Inf.	4	Inf.	1
h =100	5	2	1	0
h =50	0	0	0	0

Inf.: vibrator could not solve the jamming problem

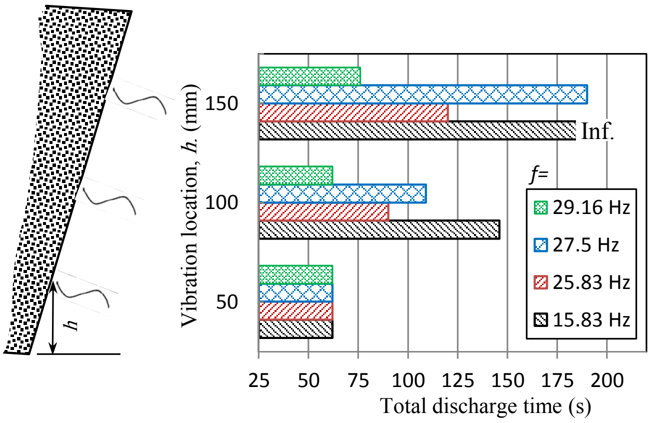

Fig. 7. The silo discharge time for different locations and frequencies of the vibrator.

discharge time. To obtain the instant discharge rate, the local slopes of the graphs belonging to different working conditions were extracted. Interestingly, it was revealed that although the vibrator was installed at different locations and worked at various frequencies, no considerable difference ocurred between the local slopes of the graphs. For example, the graphs related to a few working conditions are plotted in Fig. 8. It is observed that the local slope of all graphs are close to each other at values of approximately 0.28 kg/s. This means that although vibration condition influenced the total discharge times (the average discharge rate), it did not considerably affect the instant discharge rate.

3.3. Dimensionless acceleration

As mentioned previously, the vibrator was installed at different locations on the silo walls. There is a question on how the location of the vibrator influences other vibration characteristics such as amplitude and acceleration peaks. The amplitude peak was extracted from the amplitude-time graphs measured by the vibrometer instrument as mentioned earlier (Fig. 5). It was observed that increasing the frequency intensified the vibration amplitude. However, the measurements revealed that the increasing trend depended on the vibrator location. To identify the vibration via a more general parameter, such as a combination of frequency and amplitude, the dimensionless acceleration is defined below and called G-force or vibration intensity (Eq. 1) [33].

$$G = \frac{A\omega^2}{g} \tag{1}$$

where A, ω and g are amplitude peak, frequency (rad/sec), and gravity, respectively. As depicted in Fig. 9, dimensionless acceleration increased by increasing the vibration frequency. In fact, an increase of the frequency causes increasing vibration force as related to the centrifugal force of the vibrator rotating mass. This explains the exhibited dependencies of the amplitude and acceleration on the frequency. However, the graphs show that the increasing trends for frequency vary with the location of the vibrator. It is noted that when the vibrator moves to a higher location, amplitude and consequently G-force are reduced. From a practical viewpoint, when the vibrator is installed at an upper location, vibration energy is distributed on a larger region. This may be the reason for the dependency of the vibration characteristics on the vibrator location.

To clarify a probable relationship between discharge time and G-force, relevant data from the smallest to the largest G-force are presented in Table 2. Roughly speaking, it is observed that by increasing G-force, the discharge time is decreased, nevertheless, a few

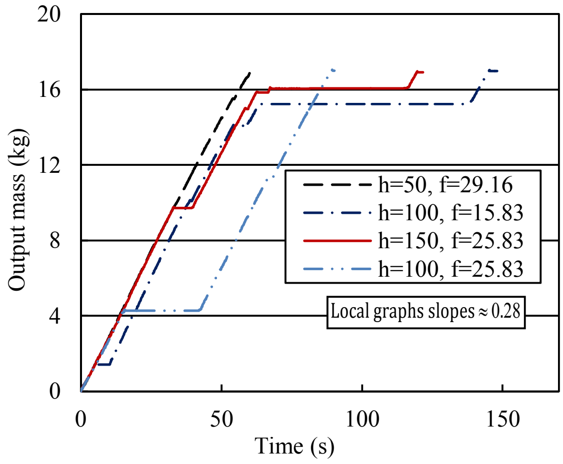

Fig. 8. Output mass-time graphs and instant discharge rate for different test conditions.

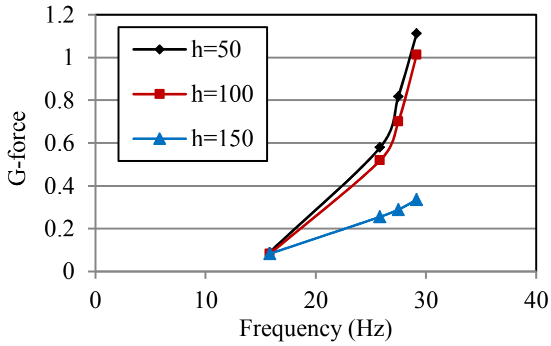

Fig. 9. Variations of the dimensionless acceleration (G-force) versus the frequency for three vibrator locations.

exceptions exist belonging to cases where the vibrator was installed near to the outlet. For instance, in Case 1 where the vibrator is close to the outlet, although G-force is low the total discharge time is short. Therefore, it can be concluded that the position of the vibrator is the most important parameter that affects its anti-jamming efficiency.

The results obtained here are in agreement with those reported in a few references. For example, similar to what was reported in [30], we observed that although vibration could solve the jamming problem it did not influence the instant discharge rate. However, the location of the vibrator was not studied in that reference. Regarding the importance of the vibrator location, our observations are in agreement with theoretical results reported in [35,36]. Nonetheless, the literature review showed that shaking the entire body of the container might have negative effects on the discharge rate which was attributed to decreasing of the bulk density due to the vibration acceleration [27-29,32]. However, we did not observe such adverse contribution for the vibration to the discharge rate. This can be due to the fact that we applied local vibration that affects only the particles near the vibrator but not all particles inside the silo. In shaking the whole body of the silo, all particles move up during the half cycle of the harmonic motion, while in local vibration a limited region of the wall is moved affecting only a group of particles. Based on the theoretical results in [34], moving a part of the hopper wall can change the stress orientation in the granular materials leading to instability of the arches and consequently allowing the flow. This idea can also explain our obtained experimental results related to the effects of the local vibration.

Table 2 Experimental data besides sorting the dimensionless acceleration (G-force).

Case #	*h* (mm)	f (Hz)	*G*-force	Discharge time (Sec.)
5	100	15.83	0.081	146
9	150	15.83	0.082	Inf.
1	50	15.83	0.090	62
10	150	25.83	0.255	120
11	150	27.5	0.289	Inf.
12	150	29.16	0.335	76
6	100	25.83	0.518	90
2	50	25.83	0.579	62
7	100	27.5	0.699	109
3	50	27.5	0.817	62
8	100	29.16	1.0133	62
4	50	29.16	1.112	62

4. Conclusion

The effects of different vibration parameters on the silo discharge were studied experimentally. No absolute agreement was observed in the open literature about the impacts of vibration on silo flow. To enhance the knowledge on this issue, the effects of different parameters consisting of vibration frequency, vibrator location and dimensionless acceleration on the silo flow were investigated in the current research. Based on the obtained results, vibrator location was the most important factor affecting its anti-jamming efficiency. In this regard, it was proved that when the vibrator location approached the outlet of the hopper, its positive effect on anti-jamming was considerably improved. This can be due to the fact that when the vibrator is installed at a lower location near the outlet, the influenced area is decreased, and the vibration energy is distributed on a smaller region. Therefore, energy density in the region near the outlet is amplified so that it can break aggregate arches, leading to elimination of the blockage. It was concluded that the vibrator can reduce the number of outflow interruptions leading to a decrease of the total discharge time and increasing the average discharge rate. Nevertheless, the vibration did not noticeably influence the instant discharge rate. It was revealed that when the vibrator was close to the outlet, the frequency did not noticeably influence the vibration effects. Nonetheless, it was concluded that when the vibrator was far from the hopper outlet, increasing the frequency enhanced its anti-jamming impact. Moreover, to identify the vibration via a more general factor, dimensionless acceleration (G-force), as a combination of the frequency and amplitude, was defined. Although the vibration efficiency was related to this factor, no perfect relationship was observed.

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